Index Matching in Pyrex

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In this lab index matching of Pyrex with mineral oil was used to measure the absorption of different coloured Pyrex tubes. These were compared with non index matched measurements in air and water. The mineral oil data was fit to a theoretical model and was compared with the intensity of transmission from the air and water.

I. INTRODUCTION

Index matching involves placing a sample in a material with the same index of refraction as the sample. Reflection and refraction effects can be minimized to measure only the absorption of a sample. This is a classic physics demonstration where a glass rod is placed in oil rendering the rod under the surface invisible. If a coloured rod is used the edges of the tube disappear and only the colourant of the tube is visible. This can also be used as an analogue of systems where refraction and reflection are not present such as a CT or X-ray scan.

This lab uses these properties to measure the absorption of different colours of glass. The glass is placed in an index matched material so that only the absorption of the colouring in the glass affects the intensity of transmitted light.

II. Theory

When a beam of light is shone through a transparent sample the effects of reflection, refraction, and absorption diminish the amount of light measured on the other side. Refraction bends the beam as it passes through a boundary between materials according to Snell's law,

$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t). \tag{1}$$

where θ_i is the angle of the incident beam, θ_t is the angle of the transmitted beam, and n_1 and n_2 are the indexes of refraction of each material. When the materials have equal indexes of refraction $(n_1 = n_2)$ then the incident angle and the transmitted angle will also be equal $(\theta_i = \theta_t)$.

Reflection also occurs at a boundary between materials. It follows the simple reflection law $\theta_i = \theta_t$. The amount of reflection can be quantified by the reflection coefficient r, which is the ration between the incident and reflected electric fields. Fresnel's sine and tangent laws write the reflection coefficient for polarization in the plane of incidence (p-polarization), and perpendicular to the plane of incidence (s-polarization):

$$r_s = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_r)}, \tag{2}$$

$$r_s = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_r)}. \tag{3}$$

When the indexes of the two materials are the same both r_p and r_s are 0 and no light is reflected.

The other effect that is relevant is absorption, as the light propagates through a material some will be absorbed. This causes an exponential decay of the light intensity from the intimal intensity I_0 based on the distance traveled through the absorbing material Δx and the attenuation coefficient μ:

$$I = I_0 e^{-\mu \Delta x}. \quad (4)$$

When the light passes through multiple materials these effects are summed together,

$$I = I_0 e^{-\mu_1 \Delta x_1 - \mu_2 \Delta x_2 - \ldots - \mu_i \Delta x_i}. \quad (5)$$

For this lab clear mineral oil was held in a clear container and the coloured Pyrex tubes were placed in it, both of which have indexes of refraction of around 1.48 ± 0.01; the only absorption is that of the tube. This was modeled as a piecewise function $f(x)$ for the regions where the light does not pass through the tube, where it passes through one edge, and where it passes through the back and front:

$$f(x) = \begin{cases} e^{-2\mu\left(\sqrt{r_0^2 - x^2} - \sqrt{r_i^2 - x^2}\right)} & x \le r_i \\ e^{-2\mu\sqrt{r_0^2 - x^2}} & r_0 > x > r_i. \\ 1 & x \ge r_0 \end{cases} \quad (6)$$

Where x is the distance from the centre of the tube, μ is the attenuation coefficient, r_o is the outer radius of the tube, and r_i is the inner radius.

Water and air have different indexes of refraction than the Pyrex tubes and the intensity will be highest on the outside of the tube, decrease to around 0 across the edge of the tube and then increase in the middle of the tube where normal incidence would allow for light to be transmitted. The height of this transmitted spike will be dependant on how much the chosen tube absorbs the light used with higher absorption lowering the peak. The peak will be higher and wider in water than in air; water has an index of refraction of 1.33 bending the refracted beam less.

III. Method

The experimental apparatus consisted of a 405 nm violet laser, a 0.15 mm slit to narrow the beam, the sample container on a moving stand, a 4.25 mm diameter aperture, finally a photodiode was used to measure the optical power. The Newport 818-SL photodetector was used connected to a Newport 815 digital power meter which measures the optical power.

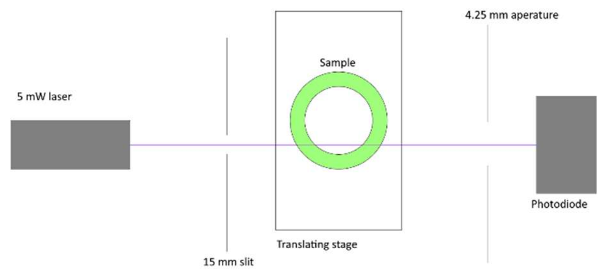

FIG. 1. Diagram of the setup used for this lab, shown with the green tube as the sample.

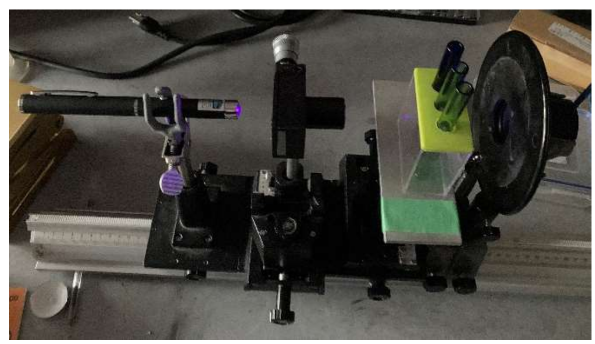

FIG. 2. A picture of the experimental setup with the laser turned on. From left to right the laser, slit, sample container, aperture, and photodetector are shown.

The sample tubes were green, blue, and purple Pyrex glassblowing tubes. Each tube was tested in mineral oil for the index matching component, as well as water and air. The Pyrex had an index of refraction of 1.470, and mineral oil has one of 1.489 which are close enough to observe index matching effects.

Tube	Inner radius	Outer radius
Green	3.4 mm	5.1 mm
Blue	3.4 mm	5 mm
Purple	3.4 mm	5.1 mm

Table 1: the inner and outer radii of each of the tubes; all measurements had the same ± 0.1 mm uncertainty from the calipers used to take measurements.

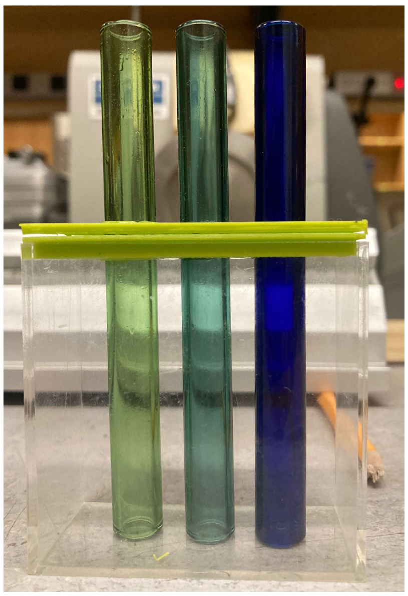

FIG. 3. The sample container with all 3 tubes in it in air.

Data was taken by placing the sample tube in the container and moving the translating stage in increments of 0.5 mm over a range of 15 mm recording the optical power for each increment.

The resulting data was normalized using the outer edges of the data where no absorption occurred to an average of 1. The theoretical model in (6) was fitted using a regression performed with a Grey Wolf algorithm on the distance squared from the theoretical curve to centre the experimental curve and determine μ. The variation on μ was determined using the uncertainty on the radii of each tube, all possible combinations of max and min radius were tested and the maximum deviation was reported as the uncertainty.

The air and water tests were used to qualitatively assess the impact of index matching when compared with the oil. As well as to show the effects of different indexes of refraction on the amount of transmitted light.

IV. Results

Figures 4-13 show the intensity curves of each colour tube in air, water and mineral oil measured across the tube as the distance from the centre. The first 3 are air, the next 3 are water and the final 3 are in mineral oil. These graphs show the normalized intensity of the transmitted light and the distance from the centre of the tube.

In air and water when the laser hits the edge of the tube it quickly refracts away dropping to 0 due to the reflection and refraction effects. When normal incidence on the tube is reached in the middle there is a spike in intensity when refraction disappears, however this is not the full intensity, there are still reflections and absorption which impact the transmitted intensity.

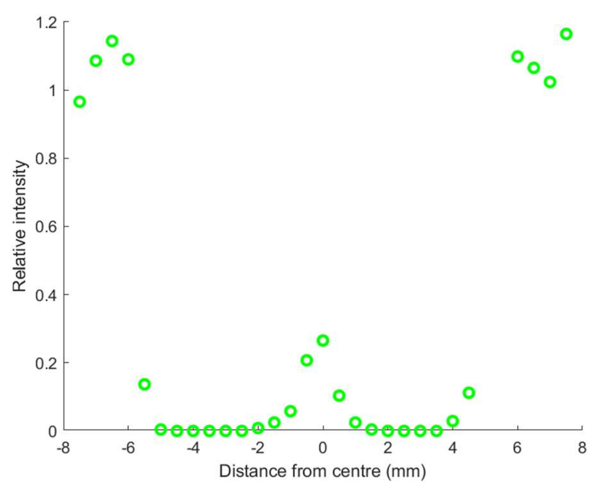

FIG. 4. The green tube in air.

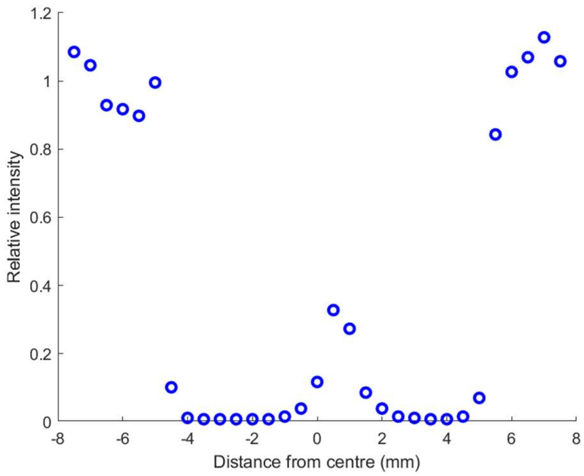

FIG. 5. The blue tube in air.

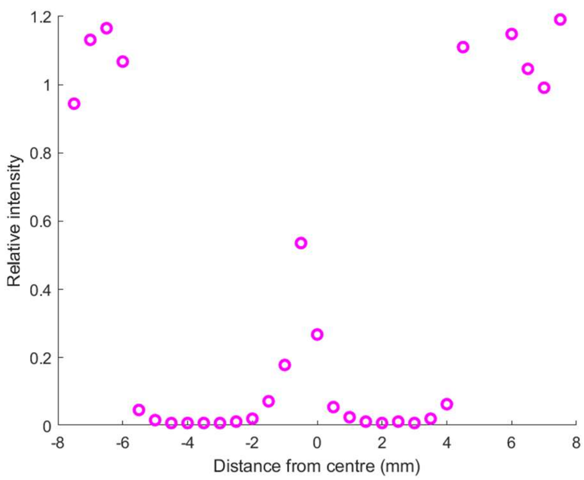

FIG. 6. The purple tube in air.

The data for water looks similar to that for air. The spike in the middle from the normal incidence is higher than in air due to the index of refraction in water being closer to that of Pyrex which makes the reflection effects are smaller and more of the light is transmitted through the middle with a slightly wider spread.

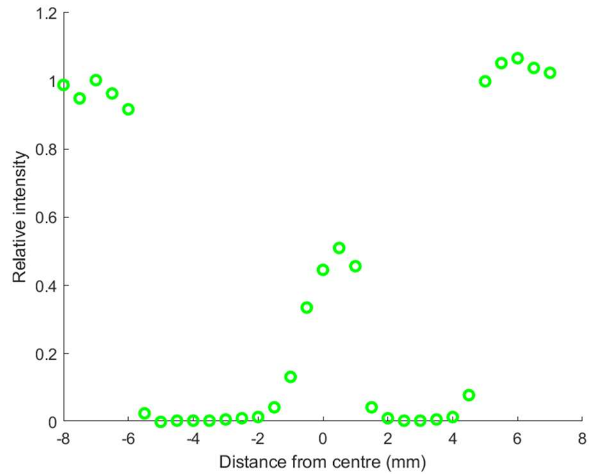

FIG. 7. The green tube in the water.

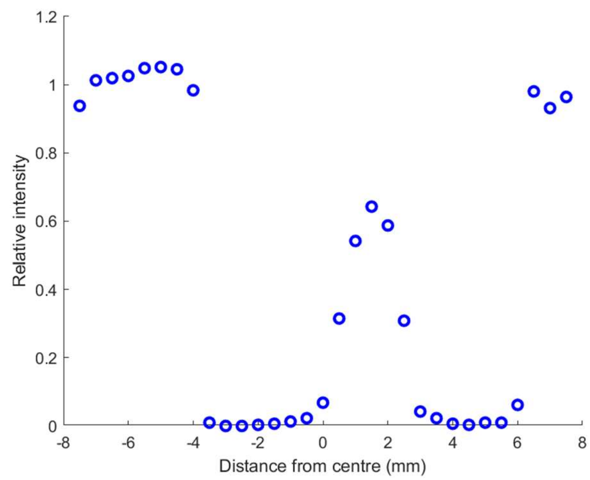

FIG. 8. The blue tube in water.

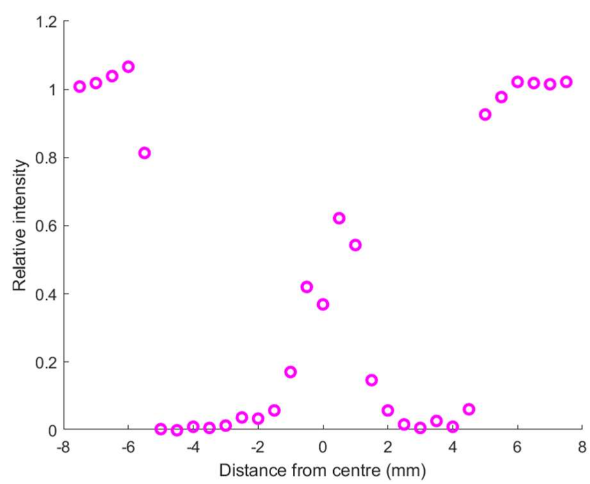

FIG. 9. The purple tube in water.

In mineral oil the index matching produces a distinctly different graph, with only the absorption relevant. The intensity drops off quickly through the edge of the tube before reaching the inner wall where the distance travelled through the absorbing Pyrex is largest. Then the intensity climbs as the towards the middle where the distance through the Pyrex is the shortest before mirroring this behaviour on the other side to produce the distinctive 'vampire teeth' curve. For figures 11-13 the experimental data points are shown in the colour of the tube as with the air and water curves above, and the theoretical curve is shown in orange.

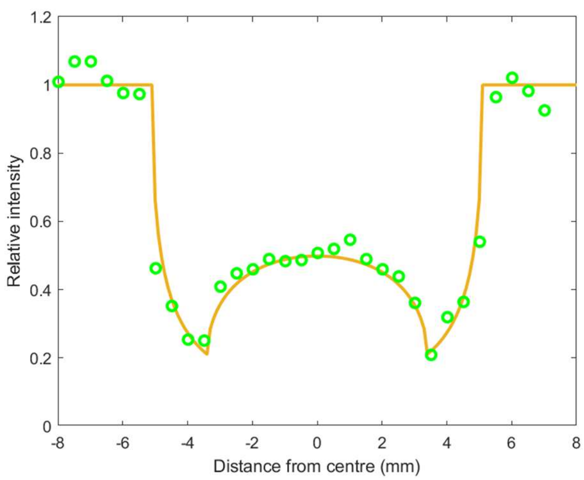

FIG. 11. The green tube in mineral oil.

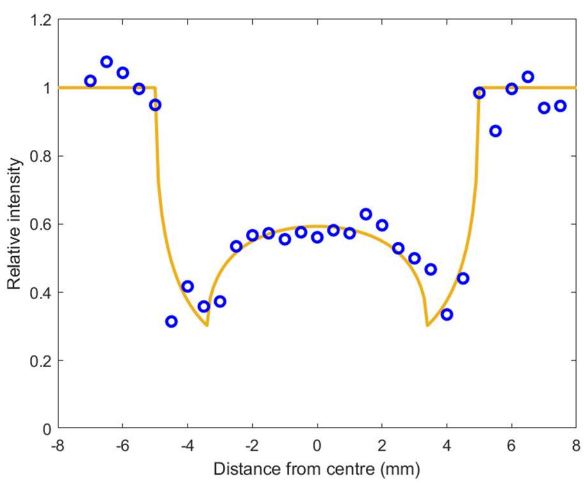

FIG. 12. The blue tube in mineral oil.

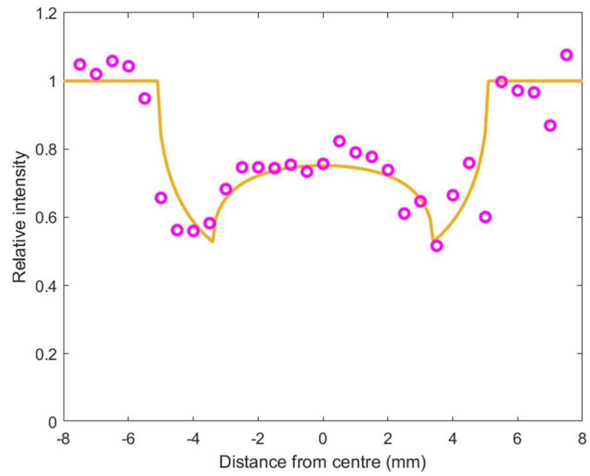

FIG. 13. The purple tube in mineral oil.

; the local minimum in the middle is higher than the green since the blue tube absorbs less of the violet light then the green one.

A regression was performed to find μ for each mineral oil curve which was: $0.205 \pm 0.005\ mm^{-1}$ for green, $0.148 \pm 0.012\ mm^{-1}$ in blue, and $0.0839 \pm 0.0006\ mm^{-1}$ in purple. The uncertainty on these values was determined be running the regression with the maximum and minimum values of the radii from the uncertainty on those measurements. These align somewhat with the literature values of $0.230 \pm 0.003\ mm^{-1}$, $0.178 \pm 0.003\ mm^{-1}$ and $0.082 \pm 0.003\ mm^{-1}$ for green, blue, and purple tubes respectively; however, our they do not fall within the margin off error of each other. For the purple tube they align, and for the green and blue our values were slightly lower than the literature ones. These discrepancies could have resulted from some of the sources of error in the discussion.

V. Discussion and Further Study

This experiment proved somewhat difficult to set up, and we encountered several difficulties before any useful data could be obtained. The first translating table used only would move in one direction, it was replaced with a moving clamp which was less precise but simpler to operate. There were also issues with stability of the laser, the clamp holding the laser pointer had to be replaced with a larger one to better stabilize the laser; however, the issue was still somewhat present due to mounting on an optical rail with clamps rather than something more stable. Another issue was with power drain on the laser, this appeared to be fixed with a battery replacement, however, even after this the power emitted by the laser appeared to drain across the multiple trials, using a bench laser would have helped eliminate this. The translating clamp we used was limited for fine control it had, the samples every half millimetre were about as small as was possible to achieve on it with any precision. Using humans to both control the stage and record the data introduces the option for errors in the movement of the stage and the recording of the data.

Computer controlling the movement of the stage and recording the data would alleviate these issues and allow more samples to be taken more quickly. Which would have helped to alleviate many of these issues.

Additional trials would have allowed μ to be better determined, comparing this with an independent measurement of the attenuation such as from spectroscopy to measure the transmission of coloured light though would help further verify. Modelling the height of the central peaks according to the attenuation coefficient would allow for more of the effects to be measured and quantified, using polarized light and this central spike the reflection coefficients may be determinable.

VI. Conclusion

Using index matching the absorption of different 405 nm light by different colours of Pyrex were measured. These curves were compared with each other, the purple tube absorbed the least of the violet light and the green absorbed the most. These curves fit the theoretical ‘vampire teeth’ model very well, and from them the attenuation coefficient μ was determined. For the tubes in air and water they produced the one spike in the middle and no transmission on either side. The height of these spikes corresponded with the absorption of each from the mineral oil.